Remember that quizzes are based on random problems from this worksheet. Please endeavor to work through these problems before the end of each discussion.

1. Find a power series representation of the function and determine the interval of convergence.

   a) \( f(x) = \frac{2}{3 - x} \).

   Answer: \( f(x) = \sum_{n=0}^{\infty} 4^n x^{2n} \) provided \( |4x^2| < 1 \). \( R = 1/2 \), \( I = (-1/2, 1/2) \).

   b) \( f(x) = \frac{5}{1 - 4x^2} \).

   Answer: \( f(x) = \sum_{n=0}^{\infty} 4^n x^{2n} \) provided \( |4x^2| < 1 \). \( R = 1/2 \), \( I = (-1/2, 1/2) \).
2. (a) Use differentiation to find a power series for

\[ f(x) = \frac{1}{(1 + x)^2}. \]

What is the radius of convergence?

*Hint:* \[ \frac{1}{(1 + x)^2} = \frac{d}{dx} \left( \frac{1}{1 + x} \right) \]

*Answer:* \[ f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} nx^{n-1} \quad \text{provided} \quad |x| < 1. \] \[ R = 1. \]
(b) Use part(a) to find a power series for

\[ f(x) = \frac{1}{(1 + x)^3}. \]

**Hint:**

\[ \frac{1}{(1 + x)^3} = \frac{d}{dx} \left( \frac{-1/2}{(1 + x)^2} \right) \]

**Answer:**

\[ f(x) = (-1/2) \sum_{n=2}^{\infty} (-1)^{n+1} n(n - 1)x^{n-2} \text{ provided } |x| < 1. \]

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(c) Use part(b) to find a power series for

\[ f(x) = \frac{x^2}{(1 + x)^3}. \]

**Answer:**

\[ f(x) = (-1/2) \sum_{n=2}^{\infty} (-1)^{n+1} n(n - 1)x^n \text{ provided } |x| < 1. \]
3. Find a power series representation for the function and determine the radius of convergence.

\[ f(x) = \ln(5 - x). \]

\[ \text{Hint: } - \frac{1}{(5 - x)} = \frac{d}{dx} (\ln(5 - x)) \]

\[ \text{Answer: } f(x) = \ln(5) - \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n + 1)5^{n+1}} \text{ provided } |x| < 5. \quad R = 5. \]

4. Find a power series representation for the function and determine the radius of convergence.

\[ f(x) = \arctan(2x) \]

\[ \text{Hint: } \frac{2}{1 + 4x^2} = \frac{d}{dx} (\arctan(2x)). \]

\[ \text{Answer: } f(x) = 2 \sum_{n=0}^{\infty} (-4)^n \frac{x^{2n+1}}{2n + 1} \text{ provided } |x| < \frac{1}{2}. \quad R = 1/2. \]