1. Determine whether the series is convergent or divergent.

a) \[ \sum_{n=1}^{\infty} \frac{\sin(2n)}{1 + 2^n}. \]

Absolutely convergent and hence convergent by the Comparison Test and Geometric series Test.

b) \[ \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}. \]

Convergent by the Alternating series Test.
c) \( \sum_{n=1}^{\infty} ne^{-n^2} \).

Convergent by the Integral Test or Ratio Test.

d) \( \sum_{n=1}^{\infty} \frac{2^n}{n!} \).

Answer: Convergent by the Ratio Test.
e) \( \sum_{n=1}^{\infty} \frac{n + 1}{n\sqrt{n}} \).

Divergent by the Comparison Test and the p-series Test or Limit Comparison test.

f) \( \sum_{n=1}^{\infty} (e^{-n} - e^{-(n+1)}) \).

*Hint: This is a telescoping sum. Consider* \( s_n = \sum_{i=1}^{n} \left( \frac{1}{e^i} - \frac{1}{e^{(i+1)}} \right) \).

Convergent because the sequence of partial sums is convergent.