Remember that quizzes are based on random problems from this worksheet. Please endeavor to work through these problems before the end of each discussion.

1. Determine whether the series converges or diverges.

   a) \( \sum_{n=1}^{\infty} \frac{n}{2n^3 + 1} \).

      *Hint: \( 2n^3 + 1 > 2n^3 \) for all \( n \geq 1 \) so if you flip the quantities on both sides of your inequality, you can easily get a larger term to use for the comparison test. Otherwise, the limit comparison test would apply too.*

   b) \( \sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k^3} + 4k + 3} \).
c) \[\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{1.2}}.\]

*Hint: \(\arctan(n) < \frac{\pi}{2}\), for \(n \geq 1\). To see this, look at the graph of \(\arctan(x)\).*

d) \[\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}.\]

*Hint: use the comparison test because the limit arising from the limit comparison test is hard to evaluate. I’m not even sure how to compute it. Even if we have a minus sign in the \(n\)–th term of the series, we are always subtracting 2 regardless of the value of \(n\) so the estimate to make is obvious.*
e) \[ \sum_{n=1}^{\infty} \frac{n^2 - 5n}{n^3 + n + 1} \]

Hint: My first guess would be to use the limit comparison test. The minus sign in the numerator makes it hard for me to make an obvious estimate to apply the comparison test. When there is a minus sign between terms which depend on \( n \) and the limit \( a_n/b_n \) is easy to compute with a good choice of \( a_n \) and \( b_n \), use the limit comparison test. Don’t forget that these tricks only work for series with positive terms.
f) \[ \sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n} \]

*Hint:* \( e^{\frac{1}{n}} > 1 \) for all \( n \geq 1 \)

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g) \[ \sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{n^3 + n^2} \]

*Hint:* The limit comparison test is what I would use here since the limit is easy to compute. The comparison test can also be used but you need to shift the index to recognize the series you are using for comparison. The second option is what the solution manual does which is tricky!