Final Exam Review

1. Evaluate \( \int \frac{9}{(9 + 4x^2)^2} dx \)

2. (a) Find the first five terms of the power series representation for the function \( f(x) = e^x \cos(x^2) \).

(b) Use that to find \( f^{(4)}(0) \) (i.e. the fourth derivative of \( f \), evaluated at 0).

3. A tank initially holds 16 L of water in which 4 kg of salt has been dissolved. Brine that contains 6 kg of salt per liter enters the tank at the rate of 2 L/min, and the well-stirred mixture exits at the same rate. How much salt is in the tank after \( t \) minutes?
4. Determine whether the series \( \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2 - 1}} \) converges or diverges.

5. (a) Find the line of intersection of the planes \( x + y + 3z = 7 \) and \( x + 2z = 4 \).

(b) Find an equation of the plane that contains the line of intersection from part (a) and the point \( (2,1,3) \).

6. Evaluate \( \int_{-1}^{1} \frac{dx}{x^2 - 2x} \) or show that it is divergent.

7. Set up an integral for the area of the surface generated by rotating \( y = x^2 \), \( 1 \leq x \leq 2 \) about the line \( x = -3 \).
8. Find the radius and interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(5x - 4)^n}{n^3} \).

9. Draw the polar curves \( r = \sqrt{3} \cos \theta \) and \( r = \sin \theta \), and then find the area that lies inside both curves.

10. At what points on the curve \( x = 2t^3, \ y = 1 + 4t - t^2 \) does the tangent line have slope 1?

11. Find a vector function that represents the curve of intersection of the cylinder \( x^2 + y^2 = 16 \) and the plane \( x + z = 5 \).