

CALCULUS  
Volume by slices and  
the disk and washer methods:  
Problems  
NEW

NEW 0720-1. Let  $R$  be the region bounded by  
 $y = x + 2$  and  $x = 4$  in  $2 \leq y \leq 3$ .

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the  $x$ -axis.
- Find the volume of the solid obtained by rotating  $R$  about the  $y$ -axis.

NEW 0720-2. Let  $R$  be the region bounded by  
 $y = x^2$  and  $y = 2x$ .

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the  $x$ -axis.
- Find the volume of the solid obtained by rotating  $R$  about the line  $x = -2$ .

0720-3. Let  $R$  be the region bounded by  
 $y = \ln x$ ,  $x = 9$  and  $y = 1$ .

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the  $y$ -axis.

0720-4. Let  $R$  be the region bounded by  
 $y = \cos x$  and  $y = 0$  in  $0 \leq x \leq \frac{\pi}{3}$ .

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the  $x$ -axis.

Hint:  $\cos^2 x = \frac{1 + [\cos(2x)]}{2}$

0720-5. NEW Let  $R$  be the region bounded by

$$(x - 1)^2 + (y - 4)^2 = 9.$$

- a. Sketch  $R$ .
- b. Find the volume of the solid obtained by rotating  $R$  about the  $x$ -axis.

**Note:** This solid is called a torus. It is in the shape of a doughnut.

**Hint:** Remember that  $2 \int_{-3}^3 \sqrt{9 - u^2} du$  is known; it is the area enclosed in a circle of radius 3.

NEW 0720-6. Let  $R$  be the region bounded by

$$y = x^5 \text{ and } x = y^6.$$

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the line  $y = -1/2$ .
- Find the volume of the solid obtained by rotating  $R$  about the line  $x = -1/3$ .

NEW 0720-7. Let  $R$  be the region bounded by

$$y = x^4 \text{ and } x = y^6.$$

- Sketch  $R$ .
- Find the volume of the solid obtained by rotating  $R$  about the line  $y = -1/2$ .
- Find the volume of the solid obtained by rotating  $R$  about the line  $x = -1/3$ .

**0720-8.** Let  $R$  be the region bounded by  
 $x = 2 + e^y$ ,  $x = \sin y$  in  $0 \leq y \leq \pi/2$ .

Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating  $R$  about the line  $x = 6$ .

**0720-9.** Describe the solid of revolution whose volume is given by

$$\pi \int_1^{3/2} (9e^{2y} - 2) dy.$$

Do not evaluate this integral.

**0720-10.** Describe the solid of revolution whose volume is given by

$$\pi \int_0^{\pi/2} (4 + \cos y)^2 - 16 dy.$$

Do not evaluate this integral.

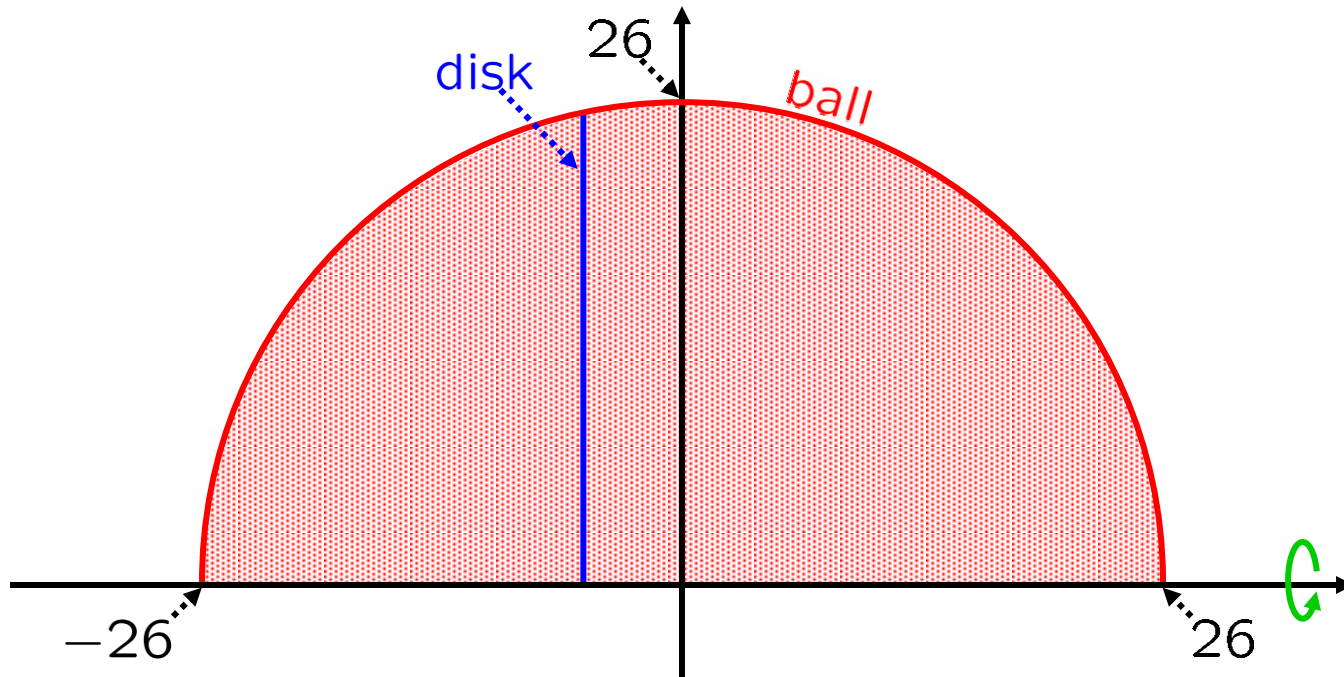
0720-11. A solid  $S$  sits above a horizontal plane  $P$ .  $\forall x \geq 0$ , let  $P_x$  be the horizontal plane that is  $x$  units above  $P$ . Suppose that  $S$  lies between  $P_1$  and  $P_2$ . Suppose, also, that  $\forall x \in [1, 2]$ , the intersection of  $S$  and  $P_x$  is the region inside a rectangle

whose base has length  $5x$

and whose altitude has length  $e^{3x^2}$ .

Compute the volume of  $S$ .

0720-12. Using the disk method, find the volume in a ball of radius 26, following the diagram shown below.





0720-13. We create a napkin holder by drilling a cylindrical hole of radius 10 through the middle of a ball of radius 26, as shown below. Using the washer method, find its volume.

