Math 8201 **Homework 12** **PJW**

Date due: December 2, 2002. There will be a quiz on this date.

Unfortunately I am behind what I had anticipated in homework assignment 11 that I would have done by 11/25/02. We have not yet done prime ideals, and so I am re-assigning the questions on that topic to homework 12. Thus the questions due for Assignment 11 are

Section 7.3 21*, 30*, 35*.

Section 7.4 6*, 8*.

Here is Assignment 12:

Section 7.4 11*, 12, 13*, 15, 23*, 24, 27, 30, 31, 37*, 38, 40

KK*. Let $\mathbb{F}_3 = \{0, 1, -1\}$ be the field with three elements, and let $f(X) = X^2 + X - 1 \in \mathbb{F}_3[X]$. Write $\alpha = X + (f) \in \mathbb{F}_3[X]/(f)$.

(a) Express each of $\alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, \alpha^7, \alpha^8, \alpha^9$ in the form $u + v\alpha$, where $u, v \in \mathbb{F}_3$. (Thus verify that $\mathbb{F}_3[X]/(f)$ is a field.)

(b) Express $\alpha^2 + \alpha^3$ as a power of α.

Section 7.5 5*