Phaselocking (and lack thereof) in a model for glacial variability

David Morawski

School of Mathematics,
University of Minnesota

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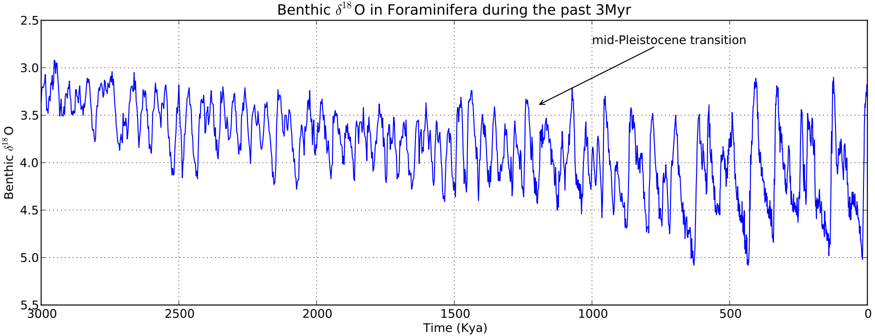
Outline

- **Background:**
  - The mid-Pleistocene transition (**MPT**)
  - Huybers' explanation for the MPT
  - Huybers' model
- **Results:** Looking at Huybers' model
  - Constant ice accumulation
  - Randomizing ice accumulation
- **Huybers' versus Hopf**

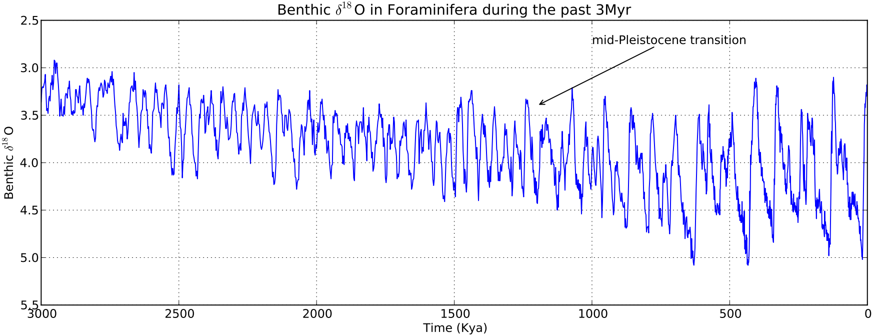
The mid-Pleistocene transition



- Before 1.2 Mya, glacial cycles had a period of 41Kyr.
- Since 1.2 Mya, the period has been 100Kyr.

Lisiecki, L.E. and M.E. Raymo. 2005. A Pliocene-Pleistocene stack of 57 globally distributed benthic D18O records. Paleoceanography, Vol. 20, PA1003, doi:10.1029/2004PA001071.

The mid-Pleistocene transition



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- Since 1.2 Mya, the period has been 100Kyr.
- **"The 100,000 year problem":** what happened?

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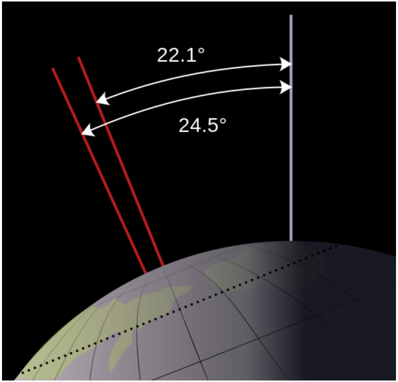
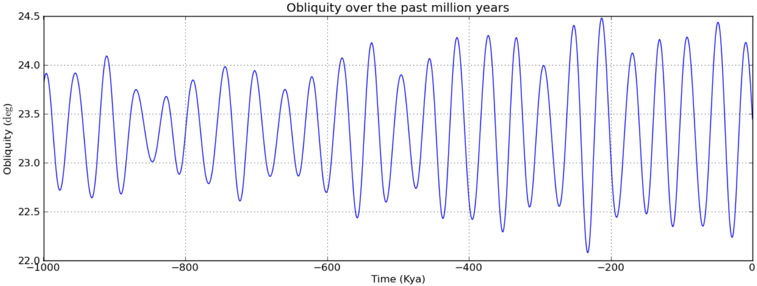
"Pacemaker of the Ice-Ages"

- Glacial cycles are driven by 3 orbital variations (Milankovitch cycles)
- The one that we care about today is **obliquity** (tilt):



Obliquity

Obliquity: 41Kyr period



Berger A. and Loutre M.F., 1991. Insolation values for the climate of the last 10 million years. Quaternary Sciences Review, Vol. 10 No. 4, pp. 297-317, 1991.

Huybers' explanation of the mid-Pleistocene transition

- Obliquity triggered deglaciations throughout the Pleistocene
- Early Pleistocene: each period of obliquity triggered a deglaciation (41Kyr)
- Late Pleistocene: deglaciations skipped two or three obliquity cycles
  - (2×41) and (3×41)Kyr cycles averaged to 100Kyr

Huybers P., 2007. Glacial variability over the last two million years: an extended depth-derived agemodel, continuous obliquity pacing, and the Pleistocene progression. Quaternary Science Reviews 26 (2007) 37-55.

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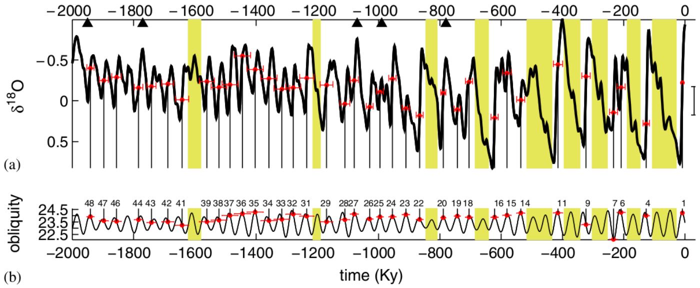

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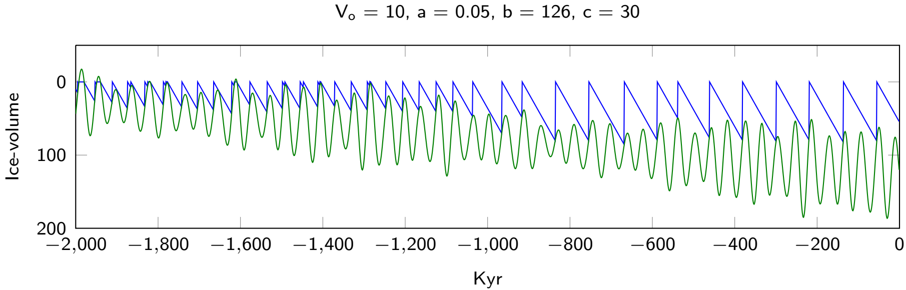
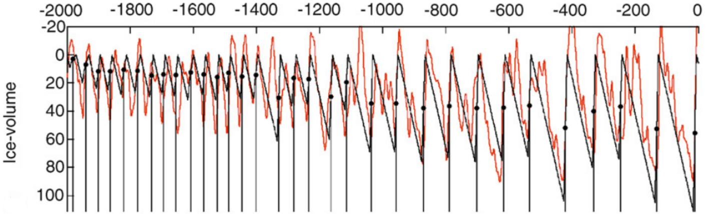
Huybers' model



$$V_t = V_{t-1} + \eta_t \quad \text{and if } V_t \geq T_t \text{ terminate,}$$
$$T_t = at + b + c\theta'_t$$

V_t:	ice volume
T_t:	threshold
η_t:	ice-accumulation
θ'_t:	obliquity (normalized)

Huybers' model



- "Selecting a slope of $a = 0.05\text{Ka}^{-1}$, an intercept of $b = 126$, and an obliquity amplitude of $c = 20$ reproduces the timing of most deglaciations over the last 2 Ma."

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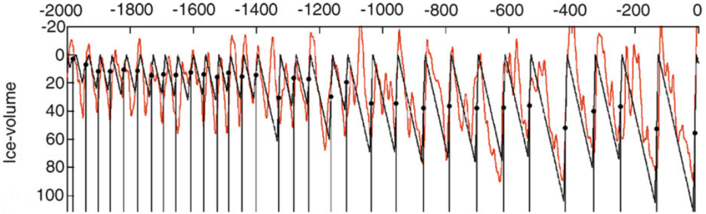
Huybers' model



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- "Exceptions are that a degleciation near 1.35 Ma is missed, the long glacial cycle at 1.6 Ma is not reproduced, . . . , and some of the smaller late-Pleistocene deglaciations."

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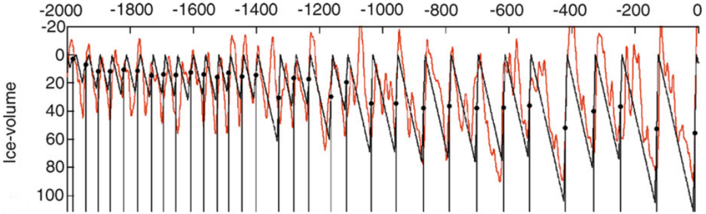

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- "Exceptions are that a degleciation near 1.35 Ma is missed, the long glacial cycle at 1.6 Ma is not reproduced, . . . , and some of the smaller late-Pleistocene deglaciations."
- Also: the amplitudes of the early-Pleistocene are too small.

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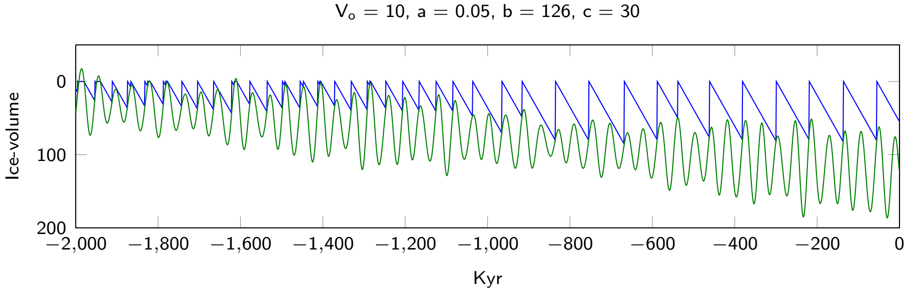
How does the model behave when varying parameters and initial condition?

Huybers' model with constant ice accumulation

Set $\eta_t \equiv 1$:



Q: For fixed b, c, how does initial ice volume v_o effect the model?

Huybers' model with constant ice accumulation

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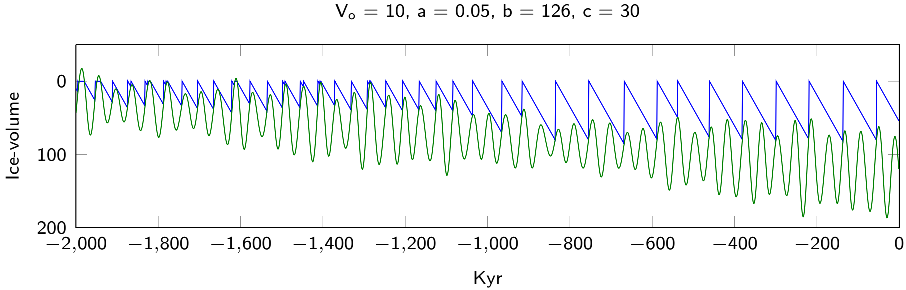

Q: For fixed b, c, how does initial ice volume v_o effect the model?
A: For most $b \in [0, 130]$ and $c \in [0, 30]$ fixed, (V_t, v_o) is eventually the same.

Huybers' model with constant ice accumulation

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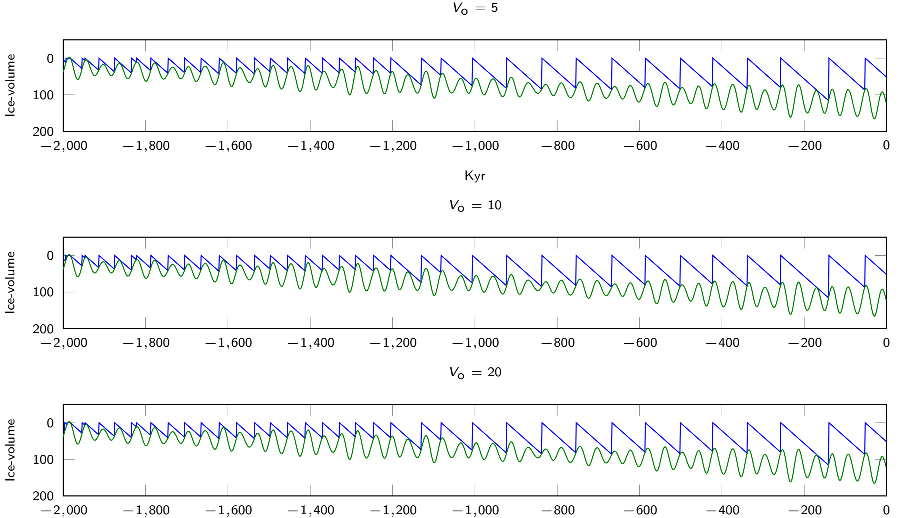
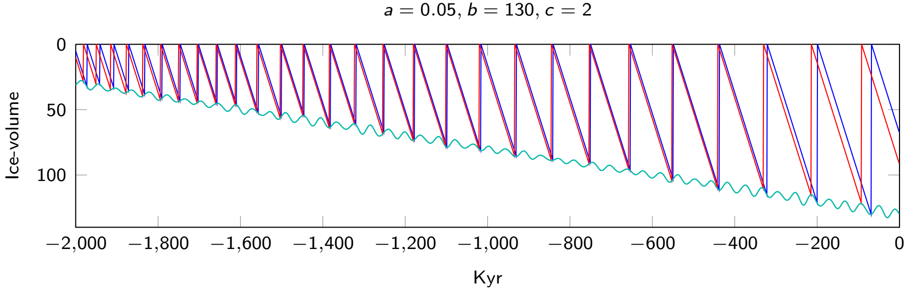


Huybers' model with constant ice accumulation

Sometimes, though, the phase is effected:



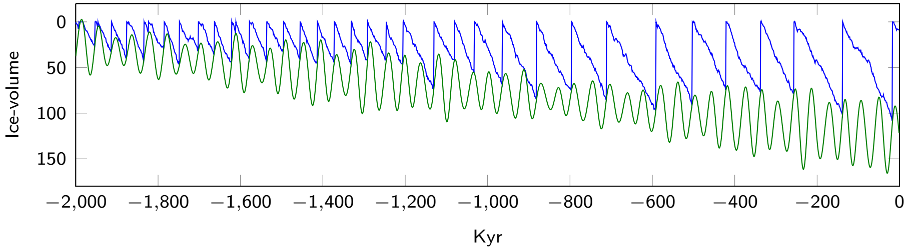
$v_o = 5$ versus $v_o = 10$

Though this is mostly for small c.

Huybers' model with random ice accumulation

Now let η_t be a random variable ($\mu = \sigma = 1$)

$V_o = 0$, $a = 0.05$, $b = 126$, $c = 20$



"The addition of a stochastic component to the model simulates the presence of weather at the highest frequencies and the myriad climatic processes not resolved by the model at longer periods."

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Huybers' model with random ice accumulation

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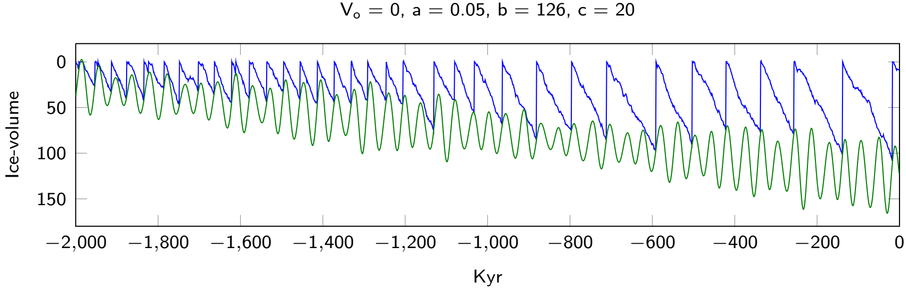

"The timing of deglaciation is still controlled by obliquity, but obliquity cycle skipping is now random so that the glacial sequence need not coincide with the δ^{18}O stack."

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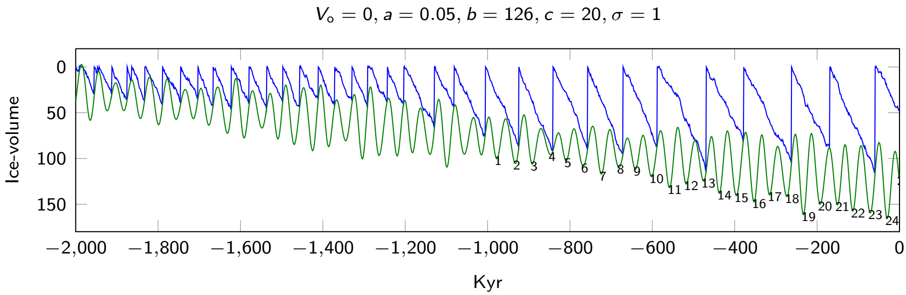
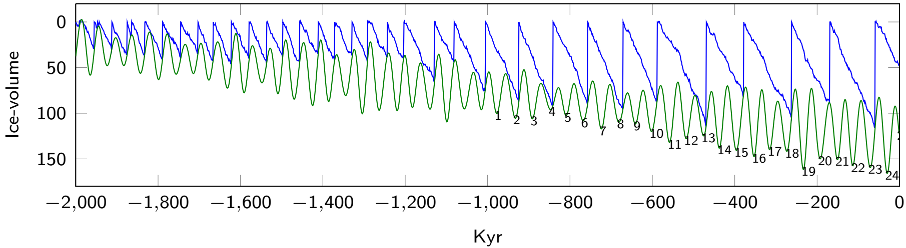
Random cycle skipping

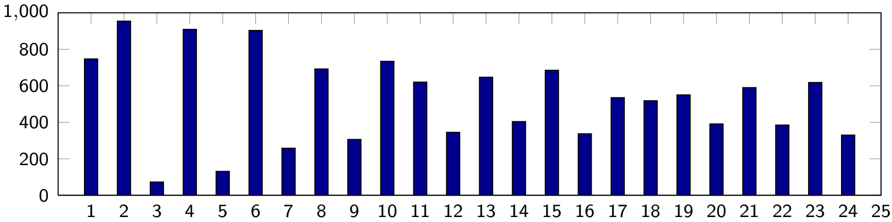


Random cycle skipping

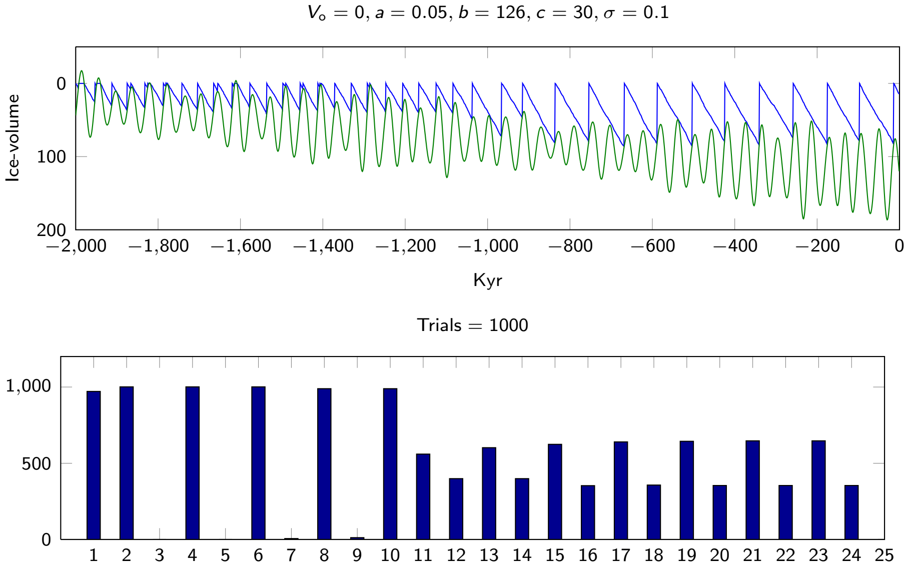
$V_o = 0, a = 0.05, b = 126, c = 20, \sigma = 1$



Trials = 1000

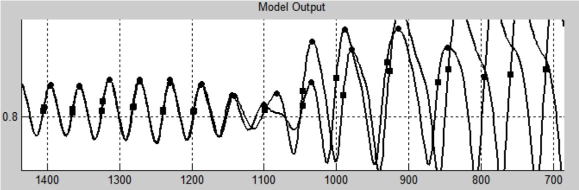


Random cycle skipping – small variance



Huybers versus Hopf

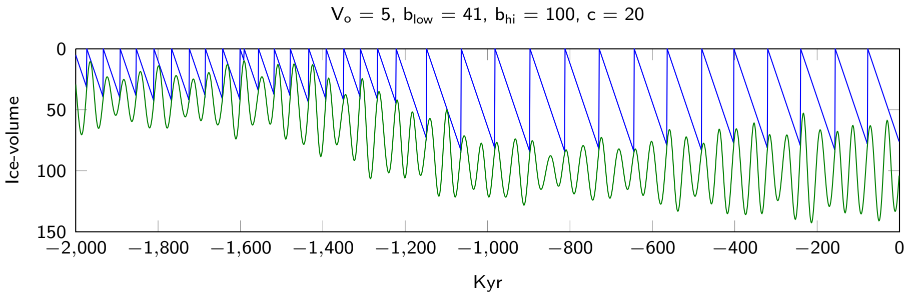
Initial Conditions of Dynamic Hopf



One example of Sensitivity to Initial Condition at the neck.

Oestreicher S., Feb 4 2014. Interpreting the Past: Modeling the 100,000 year Problem. UMN Mathematics of Climate seminar.

An altered threshold function



$$T_t = \begin{cases} b_{\text{low}} + c\theta'_t & \text{if } t \in \{\text{early Pleistocene}\} \\ b_{\text{hi}} + c\theta'_t & \text{if } t \in \{\text{late Pleistocene}\} \end{cases}$$

An altered threshold function

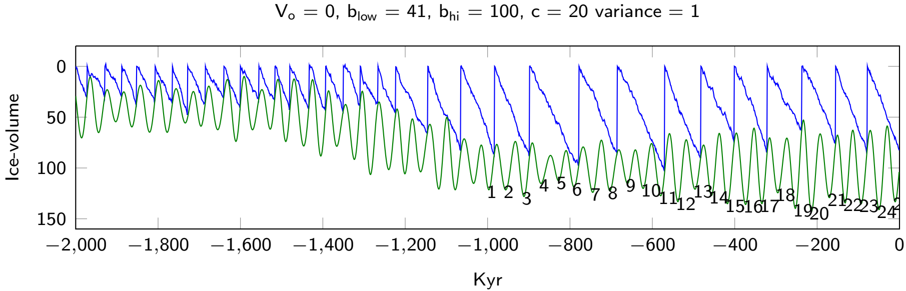
In the deterministic case: very similar.

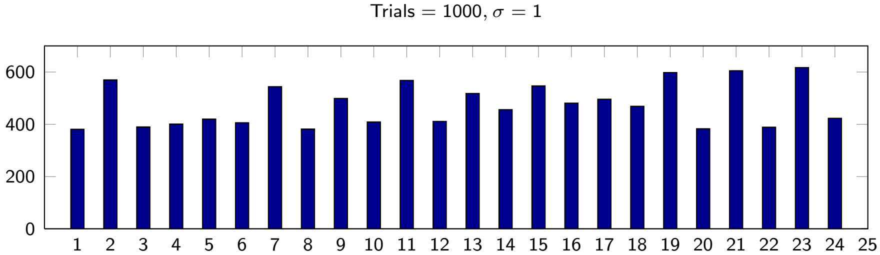
An altered threshold function

In the deterministic case: very similar.

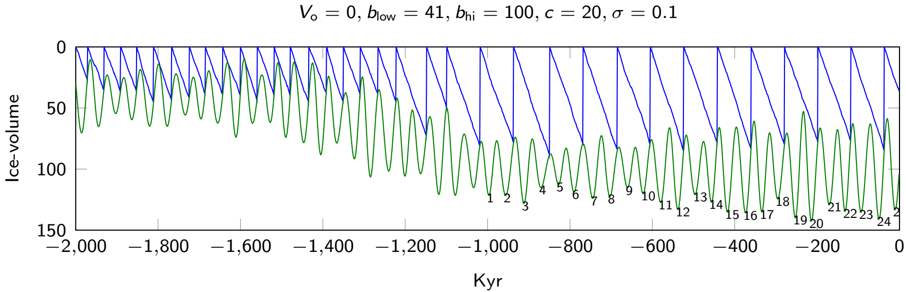
In the stochastic case: very similar.

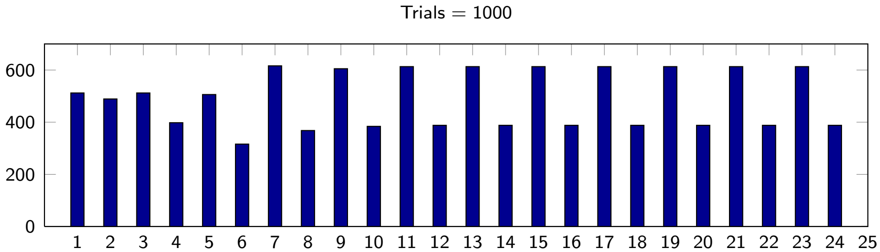
Altered threshold: stochastic





Altered threshold: stochastic, small variance





Thanks!