MATH 3283W PROFESSIONAL PROBLEM #3

CHAPTER 4, SECTIONS 4.3, 4.4 DUE APRIL 13, 2010

1. DETERMINE IF THE FOLLOWING SERIES CONVERGE OR DIVERGE. CAREFULLY SHOW ALL YOUR REASONING

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n - 2^n}$ b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}}$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^4+1}}$

2. SUPPOSE $\displaystyle\sum_{n=1}^{\infty} a_n$, $\displaystyle\sum_{n=1}^{\infty} b_n$ ARE POSITIVE SERIES SUCH THAT $\forall n \in \mathbb{N}.\ 0 < a_{2n} \leq b_{2n}$.

(1) IF $\sum b_n$ CONVERGES, DOES $\sum a_n$ CONVERGE?

(2) IF $\sum a_n$ DIVERGES, DOES $\sum b_n$ DIVERGE?

SHOW YOUR REASONING.

3 DETERMINE IF THE SERIES $\displaystyle\sum_{n=1}^{\infty} \int_n^{2n} \frac{1}{x^{7/3}}\,dx$ CONVERGES OR DIVERGES. CAREFULLY SHOW ALL STEPS IN YOUR REASONING.

<u>HINT</u> REWRITE THE SERIES USING TERMS $\displaystyle\int_n^{\infty} \frac{1}{x^{7/3}}\,dx$

4 a) PROVE THAT IF THE POSITIVE SERIES $\displaystyle\sum_{n=1}^{\infty} a_n$, $\displaystyle\sum_{n=1}^{\infty} b_n$ CONVERGE, THEN $\displaystyle\sum_{n=1}^{\infty} a_n b_n$ CONVERGES

b) IF $\displaystyle\sum_{n=1}^{\infty} a_n$ POSITIVE SERIES THAT CONVERGES AND $m \in \mathbb{N}$, PROVE THAT $\displaystyle\sum_{n=1}^{\infty} a_n^m$ CONVERGES.