Part A

For problems 1 and 2 determine if the sequences converge. If they do, find their limit. If they diverge, give reasons why they diverge.

1. (a) $a_n = \frac{\ln(n)}{n^{1/3}}, n \geq 1$

   (b) $b_n = n\tan(\frac{1}{n}), n \geq 1$

2. (a) $c_n = \frac{\sqrt{n}+1}{\sqrt[3]{n}+2}$

   (b) $d_n = \frac{(\sin(n))^n}{n}$

3. Consider the recursive sequence $s_1 = \sqrt{6}$, $s_2 = \sqrt{6+\sqrt{6}}$, and in general $s_{n+1} = \sqrt{6+s_n}$. You can assume that s_n is increasing.

   (a) Prove that $\langle s_n \rangle$ is bounded above by 6.

   (b) Why does s_n converge? Compute $\lim_{n\to\infty} s_n$.

4. (a) Use Newton's method to find a recursive sequence to estimate $\frac{1}{a}$, where $a > 0$. Hint: Look at $f(x) = \frac{1}{x} - a$, where $x > 0$.

   (b) For $a = 17$, find a good first guess $x_1 > \frac{1}{17}$. Compute x_2 and x_3. Using a calculator, how accurate is x_3?

Part B

5. Suppose $\lim_{x\to 0^+} f(x) = L$ (where $x \to 0^+$ indicates that $x \to 0$ and $x > 0$). Prove that if $a_n = f(\frac{1}{n})$ then $a_n \to L$.

6. Consider the recursive sequence $s_1 = \sqrt{5}$, and $s_{n+1} = \sqrt{5s_n}$ for $n \geq 1$.

   (a) Prove that for each $n \in \mathbb{N}$, $1 \leq s_n \leq 5$.

   (b) Prove that $\langle s_n \rangle$ is increasing.

   (c) Why does $\langle s_n \rangle$ converge? Find $\lim_{n\to\infty} s_n$.

7. Let $a_n = \frac{n^2-4n-5}{n^2-2n-3}$.

   (a) Prove that $\langle a_n \rangle$ converges. Find $L = \lim_{n\to\infty} a_n$

   (b) Find n_0 such that $n \geq n_0 \Rightarrow |a_n - L| < \frac{1}{10^2}$