Homework #10 for MATH 5345H: Introduction to Topology

December 5, 2019

Due Date: Wednesday 11 December in class.

1. In a Hausdorff space X, for every $x, y \in X$, there exist neighborhoods U, V of x and y which are disjoint. If X is regular, show that there exist neighborhoods U, V of x and y whose *closures* are disjoint.

2. Show that every locally compact, Hausdorff space is regular.

3. Suppose X is a compact metric space. Show that X is second countable. (Together with the Urysohn metrization theorem and the fact that compact Hausdorff spaces are normal, this shows that a compact Hausdorff space is metrizable if and only if it is second countable.)

4. Prove that if $A, B \subseteq X$ are subsets with $\overline{A} \cap \overline{B} \neq \emptyset$, then A and B cannot be separated by a continuous function $f : X \to [0, 1]$.