

# Homework #2 for MATH 5345H: Introduction to Topology

September 14, 2018

**Due Date:** Friday 21 September in class.

1. Suppose  $A_1, A_2, A_3, \dots$  are sets such that for each  $n$ , the intersection

$$A_1 \cap A_2 \cap \dots \cap A_n$$

is nonempty. Is it always the case that the infinite intersection

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \dots$$

is nonempty? If so, prove it. If not, give a counterexample.

2. Let  $S, T$  and  $U$  be sets. Let  $f : S \rightarrow T$  and  $g : T \rightarrow U$  be functions. For each part, give a proof or a counterexample.
  - (a) If  $f$  and  $g$  are injective, must  $g \circ f$  be injective?
  - (b) If  $g \circ f$  is injective, must  $f$  be injective?
  - (c) If  $g \circ f$  is injective, must  $g$  be injective?
3. Let  $A$  be a set, and write  $P(A)$  for the *power set* of  $A$ ;

$$P(A) = \{S \mid S \subseteq A\}$$

Assuming that  $A$  has  $n$  elements, show that  $P(A)$  has  $2^n$  elements.

**Also do these problems** from Munkres' *Topology*: ch.1, §3, # 1, 4, 11.