Last week: used knowledge of universal covers to show holomorphic maps of tori $\mathbb{C}/L \to \mathbb{C}/M$ are uniquely determined by linear map $z \mapsto \gamma z$ of universal covers, with isomorphism if + only if $\gamma L = M$.

Quick finish: Every lattice $L = \langle \omega_1, \omega_2 \rangle$ is isom. to an $M = \langle 1, \tau \rangle$ with $\tau = \omega_2/\omega_1 \in \mathcal{H}$. upper half plane.

(might have to choose $-\omega_2/\omega_1$ as generator to ensure $\tau \in \mathcal{H}$)

by $\gamma = 1/\omega_1$.

So want to know when $L\langle 1, \tau \rangle \simeq L\langle 1, \tau' \rangle$

Ans: iff $\exists\ \gamma$ with $\gamma \begin{pmatrix} 1 \\ \tau \end{pmatrix} =$ gens of $L\langle 1, \tau' \rangle$

$$\gamma = c\tau' + d$$
$$\gamma\tau = a\tau' + b$$

with $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \pm 1$.

if they are to be gens for $L\langle 1, \tau' \rangle$

divide: to eliminate γ $\quad \tau = \dfrac{a\tau' + b}{c\tau' + d}$

Since τ, τ' both chosen in $\mathcal{H}$ require $\det = 1$.

Conclusion: Isomorphism classes of cx. tori are in 1-1 corresp. with points in quotient $\mathcal{H} / SL(2, \mathbb{Z})$

$F : X \to Y$ proper holomorphic map of R.S., $\deg(F) = d$.

Can we view X as a cover of Y (i.e. is F a covering map?)

Would be if F : proper local homeom.

(if $f : \underset{\substack{\text{nbhd of} \\ 0 \in \mathbb{C}}}{\mathcal{U}} \to \mathbb{C}$ with $f'(0) \neq 0$, there exists smaller open nbhd. of 0 $\mathcal{U}' \subseteq \mathcal{U}$ s.t. f is homeom. from $\mathcal{U}'$ to its image in $\mathbb{C}$ with holomorphic inverse.)

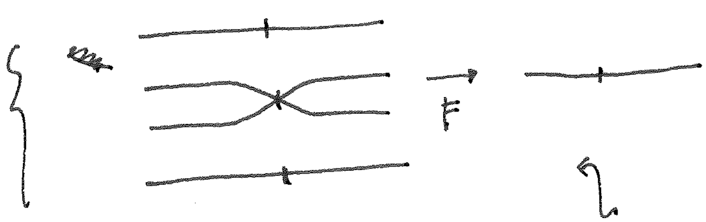
issue : ramification points.

Let $R :=$ set of ramification points $B := F(R)$ = set of branch points.

Consider $F : X \setminus F^{-1}(B) \to Y \setminus B$

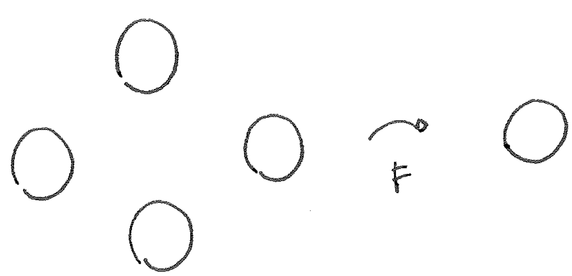
Then this is a covering map of degree d. Note : $F^{-1}(B)$ removes all points from X mapping to branch points, not just the ramification points.

Picture : 4 sheeted cover away from branch point.



this picture is bad because open sets in X, Y look like 1-dim'l subsets

Alternate picture :



By discussion last week $X \setminus F^{-1}(B)$ corresponds to index d ($d = \deg(F)$) subgroup of $\pi_1(Y \setminus B)$ (or rather conj. class of subgps)

Given set F of d elements, and fixed $f_0 \in F$,

then consider action of group π_1 on F (or equivalently a homom. $\pi_1 \to \underset{\sim}{S(F)}$)

symmetric gp. of F $\cong S_d$.

which has $\mathrm{Stab}_{\pi_1}(f_0)$: the stabilizer of f_0 under π_1 action

as subgp. of π_1

If action is transitive ($\pi_1(f_0) = F$) then $\mathrm{Stab}_{\pi_1}(f_0)$ is subgp. of index d.

In other direction, given subgp. H of π_1 of index d, then π_1

acts transitively on π_1 / H : d cosets with π_1 acting on itself by (left) multiplication in gp.

if we change $f_0 \in F$, we alter stabilizer by conjugation.

Thus for $F: X \setminus F^{-1}(B) \to Y \setminus B$, obtain a transitive rep'n

$\rho: \pi_1(Y \setminus B) \to S_d$, determined up to conjugacy. "monodromy representation"

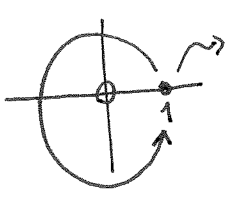
Alternate pt. of view: γ : loop in $Y \setminus B$ based at y_0.

Let's label the points in $F^{-1}(y_0)$ by $\{1, 2, \dots, d\}$. As we trace around the loop, we move the labels continuously according to preimages

then when we get back to beginning, might have changed initial labeling.

view this as effect of lifting closed loop to its covering space. endpoint of loops only live in fiber $F^{-1}(y_0)$.

Ex.: $z \mapsto z^2$. Branch point at 0.



$\{1, -1\}$

① ②

these get permuted when come around circle

$1 \to -1$ on continuous loop.

This is $\rho([\gamma]) \in S_2$.