Math 2263: Practice problems for Midterm 1

Problem 1. (15 points) Let $P = (1, 0, -3)$, $Q = (0, -2, -4)$ and $R = (2, 1, 0)$ be points.

(a) Find the equation of the plane through the points P, Q and R.

(b) Find the area of the triangle with vertices P, Q and R.

Problem 2. (15 points) Find the equation of the plane that passes through the point $(1, 0, -1)$ and contains the line $x = 2t,\ y = -2 + t,\ z = 1 - t$. What is the equation of the line of intersection of the above plane with the plane $z = 1$?

Problem 3. (20 points) A surface is created by rotating parabola $z = y^2 - 1$ about the z-axis.

(a) Sketch its surface and find its equation.

(b) Sketch the curve of intersection of this surface with the xy-plane and find its equation.

Problem 4. (15 points) Evaluate the limit

$$\lim_{(x,y)\to(1,0)} \frac{xy - y}{(x-1)^2 + 2y^2}.$$

or state that it does not exist, giving reasons.

Problem 5. (15 points) For the function $f(x, y) = (x^2 - y^2)e^{xy}$, find the second partial derivatives at $x = -1, y = 1$:

$$f_{xx}(-1, 1), \quad f_{xy}(-1, 1) \quad \text{and} \quad f_{yy}(-1, 1).$$

Problem 6. (15 points) Find parametric equation for the line through $A = (1, 2, 3)$ and $B = (0, 2, 2)$. Find the intersection between that line and the sphere of equation $x^2 + y^2 + z^2 = 8$.

Problem 7. (15 points) Find the equation of the tangent plane to the surface $z = \ln(2x + 3y)$ at the point $(2, -1, 0)$?

Problem 8. (20 points)

(a) Find the partial derivative of the function $f(x, y) = x^4 + xy - 2y^2$.

(b) Estimate $(1.01)^4 + 1.01 \times 1.98 - 2(1.98)^2$ [Hint: Use linear approximation for $f(x, y)$].

Problem 9. (20 points)

(a) Find the gradient of the function $f(x, y, z) = (x + z^2)\sin(xy)$ at the point $(x, y, z) = (1, \pi, -2)$.

(b) In which direction does the decrease of f fastest at the point $(1, \pi, -2)$?

(c) Find the directional derivative of f at the point $(1, \pi, -2)$ in the direction of the vector $\vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$.

Problem 10. (15 points) Let S be the surface consisting of all points in space whose distance to the point $(0, 0, 1)$ is $\sqrt{2}$ times their distance to xy plane. Find an equation for S and sketch the surface S.

Problem 11. (15 points) The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$ and $y = t^2 - 9$. The temperature function satisfies $T_x(2, 0) = -1$ and $T_y(2, 0) = 2$. How fast the temperature is rising on the bug's path after 3 seconds?

Problem 12. (15 points) Find the equation of the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.