Homework 6

MATH 5652 Fall 2017

October 18, 2017

1. Consider a branching process with offspring distribution given by $p_0 = \frac{1}{3}, p_1 = \frac{1}{3}$ and $p_3 = \frac{1}{3}$ and $X_0 = 1$. Determine
   (a) the expectation of X_5, the population at generation 5,
   (b) the probability that the branching process dies by generation 3, but not by generation 2, and
   (c) the probability that the process ever dies out.

2. A population consists of X_n individuals at times $n = 0, 1, 2, \ldots$. Between time n and time $n+1$ each of these individuals dies with probability p independently of the others; and the population at time $n+1$ consists of the survivors together with an independent random Poisson(λ) number of immigrants. Let X_0 have arbitrary distribution. What happens to the distribution of X_n as $n \to \infty$?

   [*Hint: Remember for a MC if $X_0 \sim \pi$ and $X_1 \sim \pi$, then π is a stationary distribution. If X_0 has* Poisson(α) *distribution for some $\alpha > 0$, what is the distribution of X_1?*]

3. A fair die with faces marked $1, 2, 3, 4, 5, 6$ is rolled repeatedly. Show that the average number of throws need to get 1000 consecutive occurrences of one of the six numbers is

$$\frac{6^{1000} - 1}{6 - 1}.$$

   [*Hint: Form a Markov chain with states $1, 2, \ldots, 1000$ with state i representing the length of the current run.*]